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Flow Of Herschel – Bulkley Fluid In An Inclined Flexible Channel Lined With Porous Material Under Peristalsis

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***Abstract*— This paper is concerned with the study of the peristaltic flow of Herschel – Bulkley fluid in an inclined flexible channel lined with porous material under long wave length and low Reynolds number assumptions. This model may be applicable to describe blood flow in the sense that erythrocytes region and the plasma regions may be described as plug flow and non-plug flow regions. The effect of yield stress , Darcy number ,angle of inclination and the index on the flow characteristics is discussed through graphs.**

Keywords: peristaltic flow,Herschel – Bulkley fluid, inclined channel.

X. INTRODUCTION

Peristaltic transport is a form of fluid transport which occurs in biological systems. This mechanism has received considerable attention in recent times in engineering as well as in medicine. It plays an indispensable role in transporting many physiological fluids in the body. Many modern mechanical devices have been designed on the principle of peristaltic pumping for transporting noxious fluids without contaminating the internal parts. Further the blood transfusion process in dialysis, the mechanism of peristalsis may be applicable since the blood flow in small blood vessels is reported to be done by peristalsis.

Latham (1966) made first experimental study of the mechanics of peristaltic transport. The results of the experiments were found to be in good agreement with the theoretical results of Shapiro (1967). Based on this experimental work, Burns and Parkes (1967) studied the peristaltic motion of a viscous fluid through a pipe and a channel by considering sinusoidal variations at the walls.

In physiological peristalsis, the pumping fluid may be a Newtonian or non – Newtonian fluid. Kapur (1985) suggested several mathematical models for pumping physiological fluids. Among these some models deal with Newtonian fluids and others with non – Newtonian fluids. Ravi Kumar et al. [ ] studied the peristaltic pumping in a finite length tube with permeable wall. Krishna Kumari et al. [ ] studied the peristaltic pumping of by considering Jeffrey model.

Scott Blair (1959) reported that blood obeys the Casson model only for moderate shear rate flows. He also reported that the assumptions included in Casson's equation are not suitable for Cow's blood and that the Herschel – Bulkley equation represents fairly closely what is occurring in the blood. Herschel – Bulkley fluid is a semi solid rather than an actual fluid.

Among models of semisolids, the Herschel – Bulkley model is preferred because it describes blood behaviour very closely and also the Newtonian, Bingham and Power – law models can be derived as special cases. Furthermore, Herschel – Bulkley fluids describe flows with a non –linear stress strain relationship either as a shear thickening fluids or shear thinning one. Since shear thinning and shear thickening fluids play an important role in biomedical engineering.

Some examples of fluids behaving in this manner include food products, pharmaceutical products, slurries, polymeric solutions and semisolid materials. Chaturani and Samy (1985) discussed the blood flow through a stenosed artery by considering blood as a Herschel – Bulkley fluid. The gastrointestinal tract is surrounded by a number of heavily innervated smooth muscle layers, contraction of these muscle layers can mix the contents of the tract and move food in a controlled manner in an appropriate direction. Epithelial cells, beneath these layers are responsible for the absorption of nutrients and water from the intestine. These layers consist of many folds and there are pores through the tight junctions of them. So the flow of fluids in different geometries in channels/tubes with porous material at the boundary is very significant in physiological applications. Vajravelu et al.[ ] considered the Herschel – Bulkley fluid in their study of peristaltic pumping of fluids.

In view of these, peristaltic flow of Herschel – Bulkley fluid in an inclined flexible channel lined with porous material is studied under long wave length and low Reynolds number assumptions. This model may be applicable to describe blood flow in the sense that erythrocytes region and the plasma regions may be described as plug flow and non-plug flow regions.

HERSCHEL – BULKLEY MODEL

The basic equations governing the flow of an incompressible Navier-Stokes fluid are the field equations

div V = 0,

$$div\sigma + \rho f = \frac{dv}{d\tau}$$

where V is the velocity, f the body force per unit mass, ñ the density, and d/dt the material time derivative. ó is the Cauchy stress defined by

$$\sigma = -pI - T$$

$$T = 2\mu D + S, \quad S = 2\eta D$$

where **D** is the symmetric part of the velocity gradient, that is,

$$D = \frac{1}{2}\left[L + L^T\right], L = grad\, V$$

Also $-\rho l$ denotes the indeterminate parof the stress due to the constraint of incompressibility, μ and **c** are viscosities.

The Herscel-Bulkely model combines the effects of Bingham and power-law behaviour in a fluid. For low strain rates $\left(\gamma < \tau_0 / \mu_0\right)$, the “rigid” material acts like a very viscous fluid with viscosity μ_0. As the strain rate increases and the yield stress threshold, τ_0 is passed, the fluid behaviour is described by a power law

$$\eta = \frac{\tau_0 + k[\dot{\gamma}^n - (\tau_0 / \mu_0)^n]}{\dot{\gamma}}$$

where k is the consistency factor and n is the power-law index.

II. MATHEMATICAL FORMULATION

Consider the peristaltic pumping of a Herschel – Bulkley fluid in an inclined flexible channel of width ‘a’. The channel is inclined at an angle θ with the horizontal. The flexible walls of the channel are coated with non erodible porous material. Suppose we have a longitudinal train of progressive sinusoidal waves on the upper and lower walls of the channel. For simplicity, we restrict our discussion to the half width of the channel as shown in Fig. 1. The region between y=0 and y = y_0 is called a plug flow region. In the plug flow region, $|\tau_{xy}| \le \tau_0$. In the region between y = y_0 and y = H, we have $|\tau_{xy}| \ge \tau_0$.

The wall deformation is given by

$$Y = H(X, t) = a + b \sin\frac{2\pi}{\lambda}(X - ct) \quad (1)$$

where b is the amplitude , λ is the wavelength and c is the wave speed.

Under the assumptions that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame (x,y) moving with the velocity c away from the fixed (laboratory) frame (X,Y) .

The transformation between these two frames is given by

x = X-ct , y = Y , u(x,y) = U (X-ct, Y) – c

v(x,y) = V(X-ct , Y) , p(x) = p(X,t) , ⊖ = ❥-Y (2)

where U and V are velocity components in the laboratory frame and u, v are velocity components in the wave frame and ⊖ and ❥ are the stream functions in the wave and laboratory frames respectively. In many physiological situations it is proved experimentally that the Reynolds number of the flow is very small. We assume that the flow is inertia free and the wavelength is infinite.

We introduce the following non-dimensional quantities in order to make the basic equations and boundary conditions dimensionless.

$$\bar{x}=\frac{x}{\lambda};\quad \bar{y}=\frac{y}{a};\quad \bar{h}=\frac{h}{a};\quad \bar{t}=\frac{ct}{\lambda};\quad \bar{\varepsilon}=\frac{\varepsilon}{a};$$

$$\phi=\frac{b}{a};\quad \bar{\tau}_0=\frac{\tau_0}{\mu\left(\frac{c}{a}\right)^n};\quad \bar{\psi}=\frac{\psi}{ac};\quad \bar{q}=\frac{q}{ac};\quad \bar{F}=\frac{fa}{\mu\lambda c};$$

$$\bar{u}=\frac{u}{c}=\frac{\partial\psi^{(1)}}{\partial\psi y_0}\quad \bar{v}=\frac{v\lambda}{ac}=-\frac{\partial\psi}{\partial x};\quad \bar{p}=\frac{pa^2}{\mu\lambda c};$$

where $\bar{\mu}$ and $\bar{v}$ are velocity components in the wave frame.

Under the lubrication approach, the equations governing the motion become (dropping bars)

$$\frac{\partial}{\partial y}\left(\tau_{yx}\right)=-\frac{\partial p}{\partial x}+\eta\sin\theta \qquad (3)$$

where $\tau_{yx}=\left(-\frac{\partial u}{\partial y}\right)^n+\tau_0$ (4)

The dimensionless boundary conditions are

$\psi=0$ at $y=0$ (2) (5)

$\psi_{yy}=0$ at $y=0$ (6)

$\tau_{yx}=0$ at y = 0 (7)

$u=\frac{-\sqrt{Da}}{\alpha}\frac{\partial u}{\partial y}-1$ at $y=h(x)-\varepsilon$ (8)

III. SOLUTION OF THE PROBLEM

On solving equations (3) and (4) together with boundary conditions (5) - (8) and $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$, we get the velocity field as

$$u=P^k\left[\frac{1}{K+1}\left\{(h-\in-y_0)^{k+1}-(y-y_0)^{k+1}\right\}+\frac{\sqrt{Da}}{\alpha}(y-y_0)^k\right]-1 \qquad (9)$$

where $p=-\frac{\partial p}{\partial x}+\eta\sin\theta$: $k=\frac{1}{n}$ (10)

We find the upper limit of the plug flow region using the boundary conditions that $\psi_{yy}=0$ at $y=y_0$ so, we have, $y_0=\frac{\tau_0}{P}$ (11)

Also by using by the condition

$\tau_{yx} = \tau_{h-\in}$ at $y = h-\in$, we obtain

$$P = \frac{\tau_{h-\in}}{h-\in} \tag{12}$$

Hence $\frac{y_0}{h-\in} = \frac{\tau_0}{\tau_{h-\in}} = \tau ; 0 < \tau < 1$

Taking $y = y_0$ in (9) we get the velocity in the plug flow region as

$$u_p = \frac{p^K}{k+1}\left[h-\in -y_0\right]^{k+1} - 1 \tag{13}$$

Integrating equations (9) and (13) and using the conditions $\psi_p = 0$ and $\psi = \psi_p$ at $y = y_0$ we get the stream function as

$$\psi = -y + p^k \{\frac{1}{k+1}[h-\varepsilon-y_0]^{k+1} y - \frac{(y-y_0)^{k+2}}{k+2}\} + \frac{\sqrt{Da}}{\alpha}\frac{(y-y_0)^{k+1}}{k+1} \tag{14}$$

$$\psi_p = -y + \frac{p^k}{k+1}\left[\left(h-\in -y_0\right)\right]^{k+1} y \tag{15}$$

The volume flux ‘q’ through each cross section in the wave frame is given by

$$q = \int_0^{y_0} u_p dy + \int_{y_0}^{h-t} u dy$$

$$= -(h-\varepsilon) + \frac{p^k}{k+1}[(h-\varepsilon-y_0]^{k+1}(h-\varepsilon) - -(h-\varepsilon-y_0)^{k+1}(\frac{h-\varepsilon-y_0}{k+2} + \frac{\sqrt{Da}}{\alpha} \tag{16}$$

The Instantaneous volume flow rate $Q(X,t)$ in the laboratory frame between the central line and the wall is

$$Q(X,t) = \int_0^H U(X,Y,t)\, dY \tag{17}$$

From equation (16) we can write

$$\frac{dp}{dx} = -\left\{\frac{(q+h-\in)(k+1)(k+2)\alpha}{\left[\left[(h-\in)^{k+2}(1-\tau)^{k+1}(k+2)\alpha\right] - (h-\in)^{k+1}(1-\tau)^{k+1}\left[(h-\in)(1-\tau) + \sqrt{Da}(k+2)\right]\right]}\right\}^{\frac{1}{k}} + \eta\sin\theta$$

(18)

Averaging equation (17) over one period yields the time mean flow (time - averaged volume flow rate) $\bar{Q}$ as

$$\bar{Q} = \frac{1}{T}\int_0^T Q\, dt = q + 1 \tag{19}$$

The pumping characteristics

Integrating the equation (18) with respect to 'x' over one wavelength, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta P=-\int_0^1\left[\left\{\frac{(Q-1+h-\in)(k+1)(k+2)\alpha}{\left[(h-\in)^{k+2}(1-\tau)^{k+1}(k+2)\alpha\right]-(h-\in)^{k+1}(1-\tau)^{k+1}\left[(h-\in)(1-\tau)+\sqrt{Da}(k+2)\right]}\right\}^{\frac{1}{k}}+\eta\sin\theta\right]dx \tag{20}$$

The time averaged flux at zero pressure rise is denoted by $\bar{Q}_0$ and the pressure rise required to produce zero average flow rate is denoted by ΔP_0.

$$\Delta P=-\int_0^1\left[\left\{\frac{(-1+h-\in)(k+1)(k+2)\alpha}{\left[(h-\in)^{k+2}(1-\tau)^{k+1}(k+2)\alpha\right]-(h-\in)^{k+1}(1-\tau)^{k+1}\left[(h-\in)(1-\tau)+\sqrt{Da}(k+2)\right]}\right\}^{\frac{1}{k}}+\eta\sin\theta\right]dx \tag{21}$$

The dimensionless frictional force F at the wall across one wavelength is given by

$$F=\int_0^1 h\left(-\frac{dp}{dx}\right)dx$$

$$F=-\int_0^1\left[\left\{\frac{h(-1+h-\in)(k+1)(k+2)\alpha}{\left[(h-\in)^{k+2}(1-\tau)^{k+1}(k+2)\alpha\right]-(h-\in)^{k+1}(1-\tau)^{k+1}\left[(h-\in)(1-\tau)+\sqrt{Da}(k+2)\right]}\right\}^{\frac{1}{k}}+\eta\sin\theta\right]dx \tag{22}$$

IV. DISCUSSION OF RESULTS

The variation of pressure rise with time averaged flux is calculated from equation (20) and is shown in Fig. (2) for different values of yield stress with $\eta=3$, $Da=0.01,\alpha=0.01,\theta=0,\phi=0.6,\in=0.1$ and $\eta=0.2$. The pumping curves intersect at a point (0.5, 0.006) in the first quadrant due to variation in the parameter τ. It is found that ΔP increases with the increase in yield stress τ to the left of the point of intersection and opposite behaviour is observed to the right of this point. For a fixed ΔP, the flux increases due to an increase in the yield stress parameter τ in the first half of the channel.

The variation of ΔP with $\bar{Q}$ for different values of τ for $\theta=\pi/3$ is shown in Fig. (3). We find that the behaviour is same for horizontal and inclined channels.

The variation of pressure rise with time averaged flex is numerically evaluated for different values of Darcy number 'Da' with $n=3, \alpha=0.01, \phi=0.6, z=0.1$ and is shown in Fig. (4).It is observed that for a given Δp, the flux decreases with an increase in the Darcy number for $0<\bar{Q}<0.5$ and the opposite behaviour is observed for $0<\bar{Q}<0.5$. For a given flux $\bar{Q}>0.5$, the pressure difference ΔP decreases with an increase in Da and the behaviour is found to be opposite for $\bar{Q}>0.5$.

The variation of pressure rise Δp with the flux $\bar{Q}$ is calculated for different values of '$\in$' with $\theta=\pi/3$, $\phi=0.6$, $\tau=0.1$ and $\alpha=0.01$ and is shown in Fig. (5.) It is found that for a given '$\in$' (thickness of the porous lining), Δp decreases with the increase in the time averaged flux $\bar{Q}$. For a given $\bar{Q}$ >0.5 (approximately), Δp increases with the increasing $\in$ and $\bar{Q}>0.5$ the variation of Δp is negligible.

The variation of pressure rise with $\bar{Q}$ for different values of angle of inclination 'θ' for different values of angel of inclination 'θ' is shown in Fig (6). It is observed that for a given Δp, the flux $\bar{Q}$ increases with the angle of inclination of the channel with the horizontal. For a given $\bar{Q}$, it is found that the pressure rise increases with the thickness of the porous lining.

The variation of pressure rise with time averaged flux is calculated for different values of n (index) and is shown in Fig.(7). We observe that the pumping curves intersect at a point (0.5, 0.17) in the first Quadrant. For a given Δp, we find that the flux $\bar{Q}$ decreases with an increase in index, 'n' for $0<\bar{Q}<0.5$ hen $\bar{Q}>0.5$, the opposite behaviour is noticed. For a given flux $\bar{Q}<0.5$. The pressure rise decreases with increasing 'n'. The opposite behaviour is observed for $\bar{Q}>0.5$.

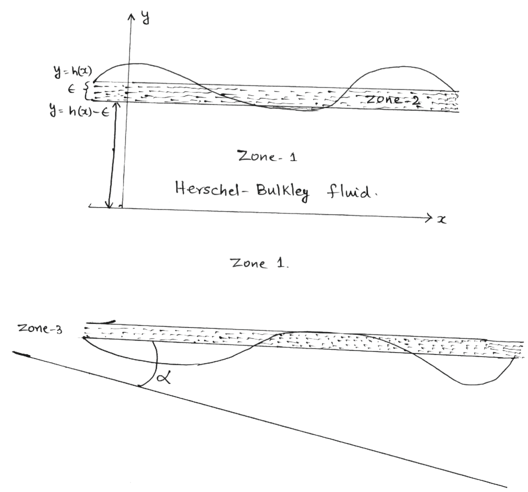

Fig.1. Physical model

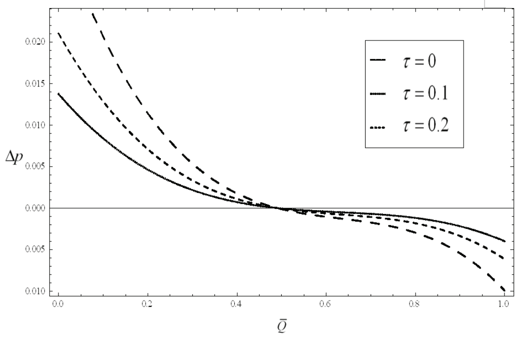

Fig 2. Variation of Δp with $\bar{Q}$ for different values of τ when n = 3,Da = 0.01, $\alpha = 0.01,\ \theta = 0,\ \in = 0.1,\ \phi = 0.6,\ \eta = 0.2$

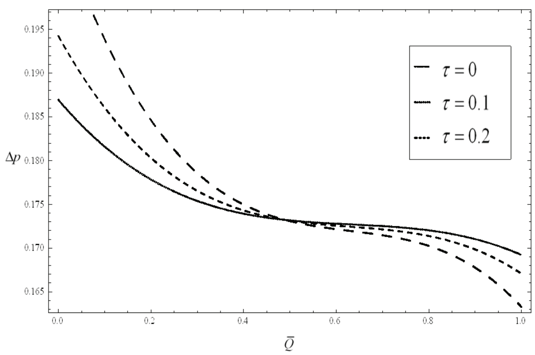

Fig 3. Variation of Δp with $\bar{Q}$ for different values of τ when n = 3,Da = 0.01, $\alpha = 0.01,\ \theta = \pi/3, \in = 0.1, \phi = 0.6, \eta = 0.2$.

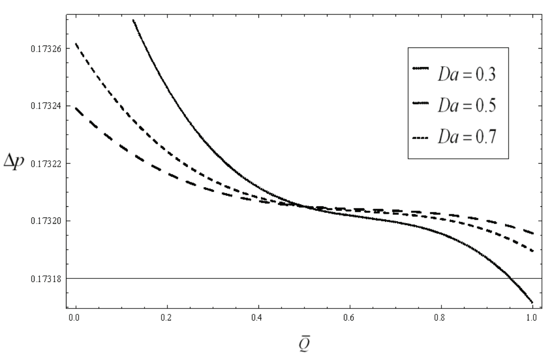

Fig 4. Variation of Δp with $\bar{Q}$ for different values of Da when n =3 ,

$\alpha = 0.01, \theta = \frac{\pi}{3}, \in = 0.1, \phi = 0.6, \eta = 0.2, \tau = 0.1$

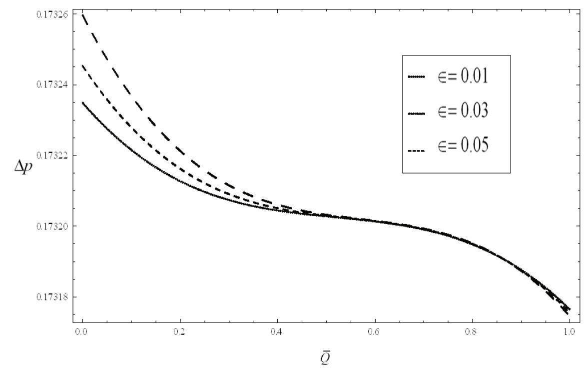

Fig 5. Variation of Δp with $\bar{Q}$ for different values of $\in$ when n=3,Da=0.3, $\alpha = 0.01, \theta = \frac{\pi}{3}, \in = 0.6, \phi = 0.6, \eta = 0.2, \tau = 0.1$

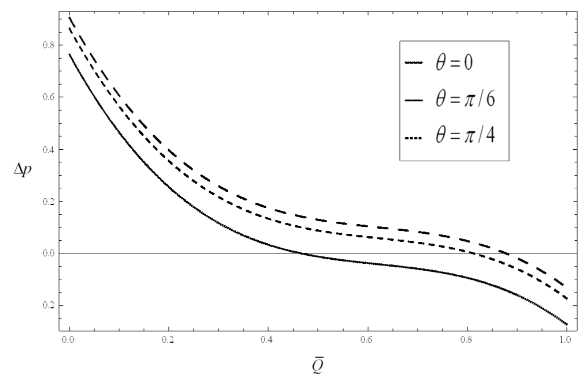

Fig 6. Variation of Δp with $\bar{Q}$ for different values of θ when n = 3, Da = 0.1, $\alpha = 0.1, \in = 0.1,\ \phi = 0.6, \eta = 0.1, \tau = 0.1$

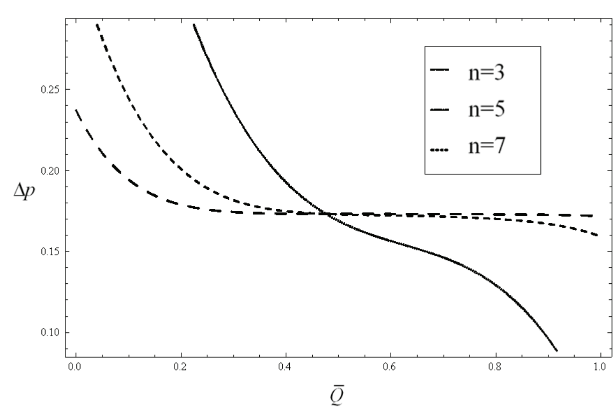

Fig 7. Variation of Δp with $\bar{Q}$ for different values of n when Da= 0.2, $\alpha = 0.1, \in = 0.1, \theta = \pi/3, \phi = 0.6, \eta = 0.1, \tau = 0.2$

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REFERENCES

[1].T.W.Latham, Fluid motion in peristaltic pumping, M.S.Thesis, MIT, Cambridge, 1966.

[2]. A.H.Shapiro,Pumping and retrograde diffusion in peristaltic waves, Proc. Workshop in ureteral reflux in children, Nat.Acad.Sci., Washington, D.C.,109-126,1967.

[3]. J.C.Burns and T.ParkesPeristaltic motion, J.Fluid Mech., 29, 731-743,1967.

[4]. J.N.Kapur, Mathematical Models in Biology and Medicine, Affiliated East – west press Pvt.Lts,New York,1985.

[5].Y.V.K.RaviKumar, S.V.H.N.KrishnaKumari, M. V. Ramana Murthy, S.Sreenadh. Unsteady peristaltic pumping in a finite length tube with permeable wall, Trans. ASME, Journal of Fluids Engineering,, Vol.32,Pages 1012011 – 1012014,2010.

[6].S.V.H.N.KrishnaKumari.P.,Y.V.K.RaviKumar, M. V. Ramana Murthy, S. Sreenadh, Peristaltic pumping of a Jeffrey fluid under the effect of magnetic field in an inclined channel, Applied Mathematical Sciences, Vol.5 No.9 , Pages 447-458,2011.

[7].S.V.H.N.Krishna Kumari.P.,Y.V.K.Ravi Kumar,M.V.Ramana Murthy,S. Sreenadh, Peristaltic transport of a Jeffrey fluid in the porous tube, ARPN Journal of Engineering and Applied Sciences, Vol.6, No.3, 2011 .

[8]. G.W.Scott Blair , An equation for the flow of blood, Plasma and Serum through glass capillaries, Nature, 183,161,1959.

[9].P.Chaturany.,R.Ponnagalar Sam.,A study of non-Newtonian aspects of blood flow through stenosed arteries and its applications in arterial diseases,Bio.rheol,22,521-531,1985.

[10].K.Vajravelu,S.Sreenadh,V.Ramesh Babu, Peristaltic Transport of Herschel-Bulkley Fluid in an inclined tube, International Journal of Nonlinear Mechanics, vol.40, Issue I,P.83,2005.

[11]. K.Vajravelu,S.Sreenadh,V.Ramesh Babu,Peristaltic pumping of a Herschel-Bulkley fluid in a Channel ,Applied Mathematics and Computation, Vol.169, ,P.726,2005.