Influence of Slip Conditions, Wall Properties and Heat Transfer on MHD Peristaltic Transport of a Jeffrey Fluid in a Non-Uniform Porous Channel

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ABSTRACT: **In this paper, we study the Peristaltic transport of magnetohydrodynamic (MHD) Jeffrey fluid in a non-uniform porous channel with the influence of slip, wall properties and heat transfer under the assumptions of long wavelength and low Reynolds number. The analytical expressions for the stream function, velocity and temperature are obtained. The results for velocity, stream function and temperature obtained in the analysis are discussed through graphs. It is noticed that the velocity and temperature decrease with increasing Jeffrey parameter λ_1. Further it is observed that the size of the trapped bolus decreases with increasing λ_1.**

Keywords:
Peristaltic pumping - Jeffrey fluid - Non-uniform porous medium - MHD - Slip flow - Heat transfer

INTRODUCTION

Peristalsis is an important mechanism for mixing and transporting fluids, which is generated by a progressive wave of contraction or expansion moving on the wall of the tube. This mechanism is found in the swallowing of food through esophagus, chyme motion in the gastro-intestinal tracts, movement of ovum in the fallopian tube and many other glandular ducts in a living body. The mechanism of peristaltic transport has been exploited for industrial applications like sanitary fluid transport, roller and finger pumps, blood pumps in heart lung machine and transport of sensitive or corrosive fluids where the contact of the fluid with the machinery parts is prohibited.

The inertia free peristaltic flow with long wavelength analysis was given by Shapiro et al., [1]. The early development on mathematical modeling and experimental fluid mechanics of peristaltic flow was given by Jaffrin and Shapiro [2]. A theoretical study of peristaltic transport of two-layered power-law fluids is made by Usha and Rao [3]. Kavitha et al., [4] studied the peristaltic flow of a Williamson fluid in an asymmetric channel through porous medium. Many researchers have contributed to the study of peristaltic transport under the effect of magnetic field and porous channel [5–10].

Peristaltic transport in non-uniform ducts is considerable interest as many channels in engineering and physiological problems are known to be of non-uniform cross-section. Srivastava et al., [11] and Srivastava and Srivastava [12] studied peristaltic transport of Newtonian and non-Newtonian fluids in non-uniform geometries. Radhakrishnamacharya and Radhakrishna Murthy [13] studied the interaction between peristalsis and heat transfer for the motion of a viscous incompressible fluid in a two-dimensional non-uniform channel. Mekheimer [14] studied the peristaltic flow of blood (obeying couple stress model) under the effect of magnetic field in non-uniform channels. He observed that the pressure rise for a couple stress fluid is greater than that for a Newtonian fluid. Also the pressure rise for uniform geometry is much smaller than that for non-uniform geometry. Hariharana [15] investigates the peristaltic transport of non-Newtonian fluid, modeled as power law and Bingham fluid, in a diverging tube with different wall wave forms.

Mittra and Prasad [16] analyzed the peristaltic motion of Newtonian fluid by considering the influence of the viscoelastic behaviour of walls. They assumed that the driving mechanism is in the form of a sinusoidal wave of moderate amplitude imposed on the flexible walls of the channel. Dynamic boundary conditions were proposed for the fluid motion due to the symmetric motion of the flexible walls which were assumed to be either thin elastic plates or membranes. Radhakrishnamacharya and Srinivasulu [17] studied the influence of wall property on peristaltic transport with heat transfer. Sobh [18] introduced slip effects on couple stress fluid. Ramana Kumari and Radhakrishnamacharya [19] investigated the effect of slip on peristaltic transport in an inclined channel with wall effects. The influence of slip, wall properties on MHD peristaltic transport of a Newtonian fluid with heat transfer and porous medium have been investigated by Srinivas and Kothandapani [20].

Among several non-Newtonian models proposed for physiological fluids, Jeffrey model is significant because Newtonian fluid model can be deduced from this as a special case by taking $\lambda_1 = 0$. Further it is speculated that the physiological fluids such as blood exhibit Newtonian and non-Newtonian behaviors during circulation in a living body. The Jeffrey model is relatively simpler linear model using time derivatives instead of convective derivatives for example the Oldroyd-B model [21]. Kothandapani and Srinivas [22] studied the peristaltic transport of a Jeffery fluid under the effect of magnetic field in an asymmetric channel. More recently, Vajravelu et al., [23] studied the influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum.

Motivated by these studies, in the present investigation we study the Peristaltic transport of magnetohydrodynamic Jeffrey fluid in a non uniform porous channel with the influence of slip, wall properties and heat transfer under the assumptions of long wavelength and low Reynolds number. The closed form of solution for the velocity, stream function and temperature are obtained. The effects of different physical parameters on the velocity, stream function and temperature obtained in the analysis are discussed through graphs.

BASIC EQUATIONS

The constitutive equations for an incompressible Jeffrey fluid are

$$\bar{T} = -\bar{p}\,\bar{I} + \bar{s} \qquad (1)$$

$$\bar{s} = \frac{\mu}{1+\lambda_1}\left(\dot{\bar{\gamma}} + \lambda_2 \ddot{\bar{\gamma}}\right) \qquad (2)$$

where $\bar{T}$ and $\bar{s}$ are Cauchy stress tensor and extra stress tensor respectively, $\bar{p}$ is the pressure, $\bar{I}$ is the identity tensor, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, $\dot{\gamma}$ is shear rate and dots over the quantities indicate differentiation with respect to time.

MATHEMATICAL FORMULATION

We consider the motion of an incompressible electrically conducting Jeffrey fluid in a two-dimensional non-uniform porous medium channel induced by sinusoidal waves propagating with constant speed 'c'. The walls of the channel are assumed to be flexible and are taken as a stretched membrane. The wall deformation $h(x,t)$ due to the infinite train of peristaltic waves is represented by

$$y = h(x,t) = d(x) + a\sin\frac{2\pi}{\lambda}(x - ct) \qquad (3)$$

where $d(x) = d + m'x,\ m' << 1,$ a is the amplitude, λ is the wavelength, *d* is the mean half width of the channel, m' is the dimensional non-uniformity of the channel.

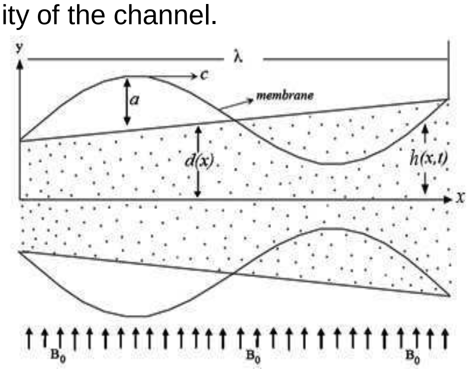


Fig. 1 Physical Model

The equations governing the flow of an incompressible Jeffrey fluid in a porous medium under the influence of a magnetic field are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (4)$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y} -\sigma B_0^2 u-\frac{\mu}{k}u \quad (5)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\partial S_{xy}}{\partial x}+\frac{\partial S_{yy}}{\partial y}-\frac{\mu}{k}v \quad (6)$$

and the energy equation [24] is

$$\zeta\rho\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) +S_{xx}\frac{\partial u}{\partial x}+S_{yy}\frac{\partial v}{\partial y}+S_{xy}\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}\right) \quad (7)$$

where

$$S_{xx}=\frac{2\mu}{1+\lambda_1}\left[1+\lambda_2\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right]\frac{\partial u}{\partial x}$$

$$S_{xy}=\frac{\mu}{1+\lambda_1}\left[1+\lambda_2\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right]\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right]$$

$$S_{yy}=\frac{2\mu}{1+\lambda_1}\left[1+\lambda_2\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right]\frac{\partial v}{\partial y}$$

Here u,v are the velocity components along x and y directions respectively, ρ is the density, μ is the coefficient of viscosity of the fluid, p is the pressure, σ is the electrical conductivity of the fluid, B_0 is the intensity of the magnetic field acting along the y-axis and the induced magnetic field is assumed to be negligible, k is the permeability of the porous medium, ζ is the specific heat at constant volume, υ is the kinematic viscosity of the fluid, κ is the thermal conductivity of the fluid and T is the temperature of the fluid.

The governing equations of motion of the flexible wall may be expressed as

$$L^*(h)=p-p_0 \quad (8)$$

where L^* is an operator, which is used to represent the motion of stretched membrane with viscosity damping forces such that

$$L^*=-\tau\frac{\partial^2}{\partial x^2}+m_1\frac{\partial^2}{\partial t^2}+C\frac{\partial}{\partial t} \quad (9)$$

Continuity of stress at $y=\pm h$ and using x momentum equation yields

$$\frac{\partial}{\partial x}L^*(h)=\frac{\partial p}{\partial x}=\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) -\sigma B_0^2 u-\frac{\mu}{k}u \quad (10)$$

$$u=\mp h_1\frac{\partial u}{\partial y} \text{ at } y=\pm h=\pm\left(d+m'x+a\sin\frac{2\pi}{\lambda}(x-ct)\right) \quad (11)$$

$$T=T_0 \text{ on } y=-h$$

$$T=T_1 \text{ on } y=h \quad (12)$$

Here τ is the elastic tension in the membrane, m is the mass per unit area, C^* is the coefficient of viscous damping, p_0 is the pressure on the outside surface of the wall due to the tension in the muscles and h_1 is the dimensional slip parameter.

It is assumed that $p_0=0$

We introduce the stream function ψ such that

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}.$$

and the following non-dimensional quantities are

$$x'=\frac{x}{\lambda},\quad y'=\frac{y}{d},\quad \psi'=\frac{\psi}{cd},\quad t'=\frac{ct}{\lambda},$$

$$S'=\frac{d}{\mu c}S,\quad \theta=\frac{T-T_0}{T_1-T_0},\quad h'=\frac{h}{d},\quad p'=\frac{d^2}{c\lambda\mu}p$$

(13)

The non-dimensional governing equations after dropping the primes, we get

$$R\delta\left[\frac{\partial^2\psi}{\partial t\partial y}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right]=-\frac{\partial p}{\partial x}+\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-M^2\frac{\partial\psi}{\partial y}-\frac{1}{Da}\frac{\partial\psi}{\partial y} \tag{14}$$

$$R\delta^3\left[\frac{\partial^2\psi}{\partial t\partial x}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x^2}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x\partial y}\right]=-\frac{\partial p}{\partial y}+\delta^2\frac{\partial S_{xy}}{\partial x}+\delta\frac{\partial S_{yy}}{\partial y}-\frac{\delta^2}{Da}\frac{\partial\psi}{\partial x} \tag{15}$$

$$R\delta\left(\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right)=\frac{1}{\Pr}\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+E\left\{\delta S_{xx}\left(\frac{\partial^2\psi}{\partial x\partial y}\right)+S_{xy}\left(\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)-\delta S_{yy}\left(\frac{\partial^2\psi}{\partial y\partial x}\right)\right\} \tag{16}$$

where

$$S_{xx}=\frac{2\delta}{1+\lambda_1}\left[1+\frac{\delta c\lambda_2}{d}\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\right]\left(\frac{\partial^2\psi}{\partial x\partial y}\right)$$

$$S_{xy}=\frac{1}{1+\lambda_1}\left[1+\frac{\delta c\lambda_2}{d}\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\right]\times\left(\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial\psi}{\partial x^2}\right)$$

$$S_{yy}=\frac{-2\delta}{1+\lambda_1}\left[1+\frac{\delta c\lambda_2}{d}\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\right]\left(\frac{\partial^2\psi}{\partial x\partial y}\right)$$

$$\frac{\partial\psi}{\partial y}=\mp\beta\frac{\partial^2\psi}{\partial y^2}\ \text{at y}=\pm h=\pm\left[1+mx+\varepsilon\sin 2\pi(x-t)\right] \tag{17}$$

$$\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-R\delta\left[\frac{\partial^2\psi}{\partial t\partial y}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right]-M^2\frac{\partial\psi}{\partial y}-\frac{1}{Da}\frac{\partial\psi}{\partial y}=\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial x\partial t}\right](h) \tag{18}$$

Further, it is assumed that the streamline value is zero at $y=0$. i.e. $\psi(0)=0$ (19)

$$\theta=0 \text{ on } y=-h \text{ and } \theta=1 \text{ on } y=h \tag{20}$$

where

$\varepsilon=\frac{a}{d}$, $\delta=\frac{d}{\lambda}$ are geometric parameters, $R=\frac{cd\rho}{\mu}$ is the Reynolds number, $M=\sqrt{\frac{\sigma}{\mu}}B_0d$ is the Hartmann number, $E_1=\frac{-\tau d^3}{\lambda^3\mu c}$, $E_2=\frac{m_1cd^3}{\lambda^3\mu}$, $E_3=\frac{-Cd^3}{\lambda^2\mu}$ are the non-dimensional elasticity parameters, $\Pr=\frac{\rho\nu\zeta}{\kappa}$ is the Prandtl number, $Ec=\frac{c^2}{\zeta(T_1-T_0)}$ is the Eckert number, $Da=\frac{k}{d^2}$ is the Darcy number, $m=\frac{\lambda m'}{d}$ is the non-uniform parameter and β is the Knudsen number (Slip parameter).

EXACT ANALYTICAL SOLUTION

Under the assumptions of long wavelength ($\delta<<1$) and low Reynolds number, from equations (14)-(18), we get

$$0=-\frac{\partial p}{\partial x}+\frac{1}{(1+\lambda_1)}\frac{\partial^3\psi}{\partial y^3}-M^2\frac{\partial\psi}{\partial y}-\frac{1}{Da}\frac{\partial\psi}{\partial y} \tag{21}$$

$$0=-\frac{\partial p}{\partial y} \tag{22}$$

Equation (22) implies $p\neq p(y)$

$$0=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+\frac{Ec}{1+\lambda_1}\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 \tag{23}$$

Elimination of pressure from equations (21) and (22), yields

$$\frac{\partial^4\psi}{\partial y^4}-N^2\frac{\partial^2\psi}{\partial y^2}=0 \tag{24}$$

where $N=\sqrt{(1+\lambda_1)\left(M^2+\frac{1}{Da}\right)}$

Equation (18) gives

$$\frac{\partial^3\psi}{\partial y^3}-N^2\frac{\partial\psi}{\partial y}=\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial x\partial t}\right](h) \quad (25)$$

The closed form solution for stream function and velocity from equation (24) using the boundary conditions (17), (19) and (25) are given by

$$\psi=-\frac{8\varepsilon\pi^3}{N^2}\left[(E_1+E_2)\cos 2\pi(x-t)-\frac{E_3}{2\pi}\sin 2\pi(x-t)\right]\times\left[\frac{\sinh Ny}{N(\cosh Nh+N\beta\sinh Nh)}-y\right] \quad (26)$$

$$u=-\frac{8\varepsilon\pi^3}{N^2}\left[(E_1+E_2)\cos 2\pi(x-t)-\frac{E_3}{2\pi}\sin 2\pi(x-t)\right]\times\left[\frac{\cosh Ny}{(\cosh Nh+N\beta\sinh Nh)}-1\right] \quad (27)$$

Substituting equation (26) into equation (23) subject to the boundary condition (20), we get the temperature as

$$\theta=\frac{BrL_1^2\left(2N^2y^2-\cosh 2Ny+\cosh 2Nh-2N^2h^2\right)}{8(1+\lambda_1)(\cosh Nh+N\beta\sinh Nh)^2}+\frac{(y+h)}{2h} \quad (28)$$

where

$$L_1=\frac{8\varepsilon\pi^3}{N^2}\left[\frac{E_3}{2\pi}\sin 2\pi(x-t)-(E_1+E_2)\cos 2\pi(x-t)\right]$$

and $Br = Ec\cdot\mathrm{Pr}$ is the Brinkman number.

The coefficient of heat transfer at the wall is given by $Z=h_x\cdot\theta_y$

$$Z=\frac{(m+2\pi\varepsilon\cos 2\pi(x-t))}{8h(\cosh Nh+N\beta\sinh Nh)^2}\left[h\frac{Br}{(1+\lambda_1)}L_1^2\left(4N^2y-2N\sinh 2Ny\right)+4\left(\cosh Nh+N\beta\sinh Nh\right)^2\right] \quad (29)$$

RESULTS AND DISCUSSION

The equation (27) gives the expression for the velocity in terms of y. Velocity profiles are plotted in figures from (2) to (8) to study the effects of different parameters such as the Jeffrey parameter λ_1, the Darcy number *Da*, the slip parameter β, the amplitude ratio ε, the non-uniform parameter m, the Hartmann number *M*, the wall tension E_1, the mass characterizing parameter E_2 and the damping nature of the wall E_3. Fig. 2 is drawn to study the effect of λ_1 on the velocity distribution *u*. We observe that the increase in λ_1 decreases the velocity.

Fig. 3 is plotted to study the effect of *Da* on the velocity. We observe that the velocity increases with increasing *Da*. Further, the permeability parameter causes to strengthen the fluid slip at the wall. Fig. 4 and Fig. 5 shows that an increase in β and ε results in the increase of velocity distribution. In Fig. 6 we find that the velocity for a divergent channel ($m>0$) is higher compared to its value for a uniform channel ($m=0$), whereas it is lower for a convergent channel ($m<0$). From Fig. 7 we see that the velocity decreases with increase of *M*. From Fig. 8 we notice that the velocity increases with increasing E_1 and E_2 and it decreases with increasing E_3.

The equation (28) gives the expression for the temperature in terms of y. Temperature profiles are plotted in figures from (9) to (16) to study the effects of the physical parameters of the problem. Fig. 9 is drawn to study the effect of λ_1 on the temperature distribution θ. It is observed that the temperature decreases with increasing λ_1. Figures 10 and 11 are plotted to study the effect of *Da* and *M* on the temperature. We observe that the temperature increases with increasing *Da* and decreases with increasing *M*. Figures 12 and 13 shows that the temperature decreases by increasing β and it increases with increasing ε.

Fig. 14 illustrates the effects of m on the temperature distribution. We notice that the amplitude of the temperature is large in case of divergent channel compared with uniform and convergent channels. Fig.15 is plotted to study the effect of Brinkman number Br on the temperature distribution. We notice that the temperature increases with an increase in Br. Fig. 16 shows that the temperature increases with increasing E_1 and E_2 and it decreases with increasing E_3.

The equation (29) gives the expression for the coefficient of heat transfer at the wall. Figures from (17) to (23) are plotted to observe the variation of heat transfer coefficient at the walls for different values of the physical parameters of interest. We observe that nature of the heat transfer is in oscillatory behavior, which may be due to peristalsis. From figures 19, 22 and 23, the magnitude of heat transfer coefficient increases by increasing *Da*, *Br*, E_1, E_2 and E_3 while from Figures 17, 18, 20, 21 and 22, it decreases by increasing λ_1, β, M and *m*.

TRAPPING PHENOMENA

The effect of slip parameter on the streamline pattern is shown in Fig. 24. We observe that the size of the trapping bolus increases with increasing slip parameter. The streamlines for Jeffrey parameter λ_1 are shown in Fig. 25. It is observed that the size of trapping bolus decreases with increasing λ_1. The streamlines for uniform and non-uniform channels are shown in Fig. 26. It is found that the size of the trapped bolus is large in the left hand of the convergent channel while it has opposite behavior for divergent channel. Further, the size of bolus is symmetric for uniform channel. From Fig. 27 it can be seen that the volume of the trapped bolus decreases with increase of M. From Fig. 28 it is clear that the trapped bolus increases in size as *Da* increases.

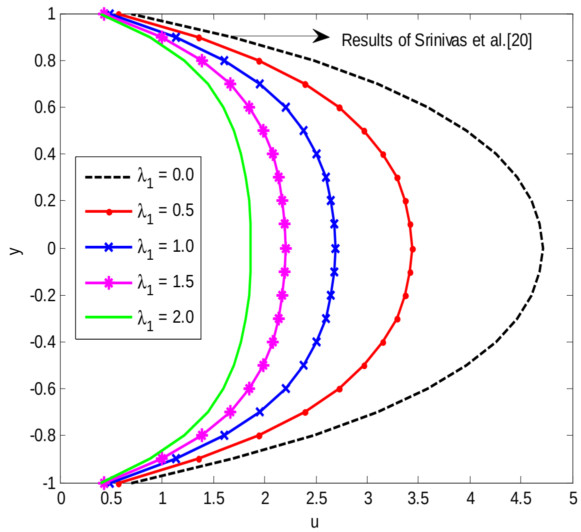


Fig.2. The variation of u with y for different values of λ_1 for fixed *E₁*=1, *E₂*=0.5, *E₃*=0.5, ε=0.1, β =0.0, *M*=2, *Da*=1, *m*=0.

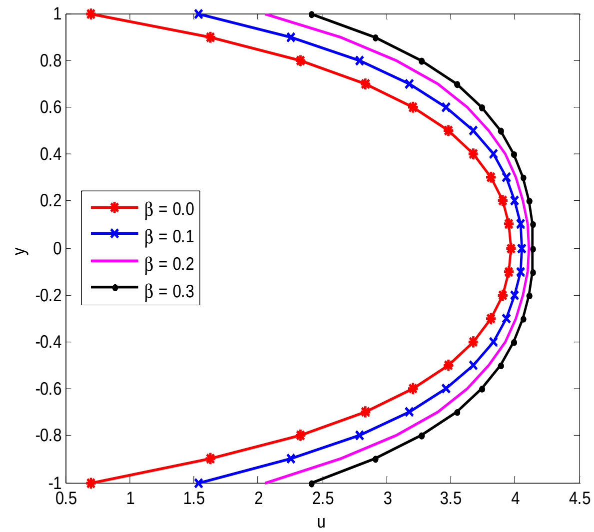


Fig.4. The variation of u with y for different values of β for fixed E_1=1, E_2=1, E_3=0.5, ε=0.1, *M*=2, *Da*=2, *m*=0, λ_1 =1.

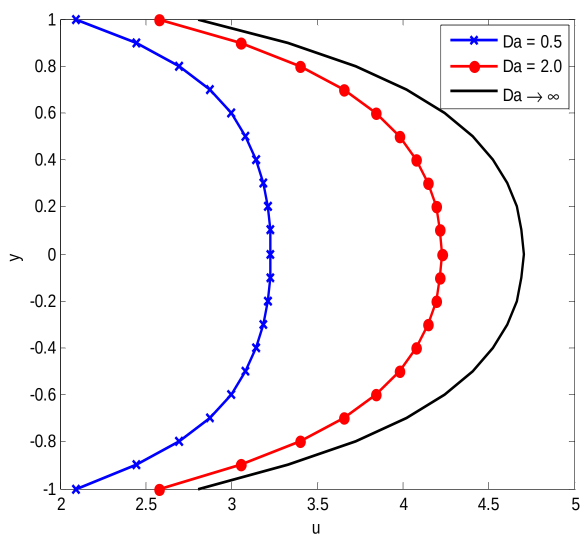


Fig.3. The variation of u with y for different values of *Da* for fixed E_1=0.5, E_2=0.5, E_3=0.1, ε=0.1, β =0.2, *M*=2, *m*=0.1, λ_1 =1.

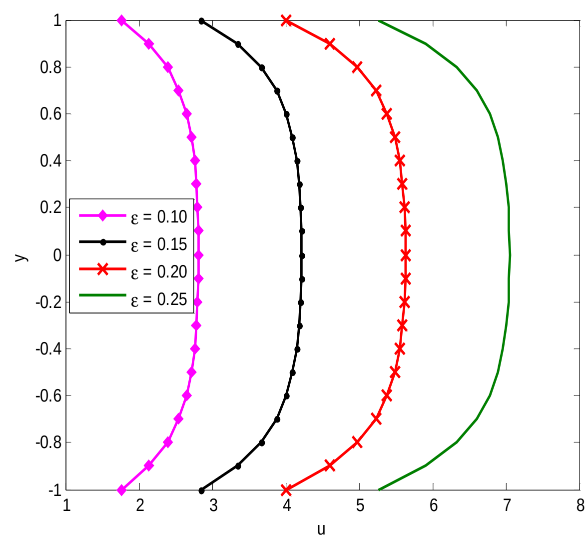


Fig. 5. The variation of u with y for different values of ε for fixed E_1=2, E_2=0.7, E_3=0.1, *M*=3, β =0.2, *Da*=2, *m*=0.1, λ_1 =1.

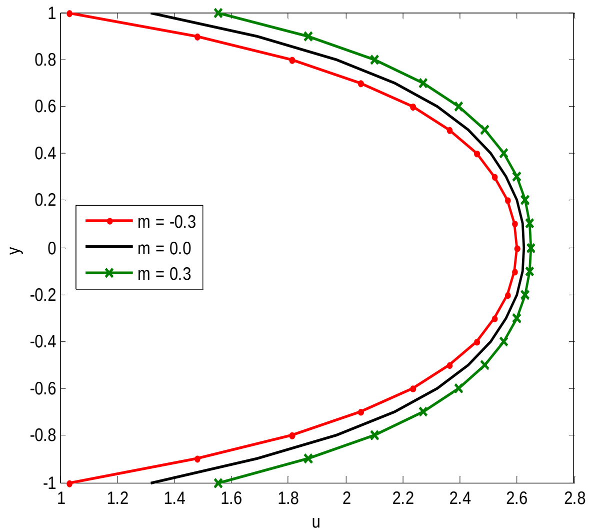


Fig. 6. The variation of u with y for different values of *m* for fixed E_1=0.8, E_2=0.5, E_3=0.5, ε=0.1, β =0.2, *M*=3, *Da*=2, λ_1 =0.5.

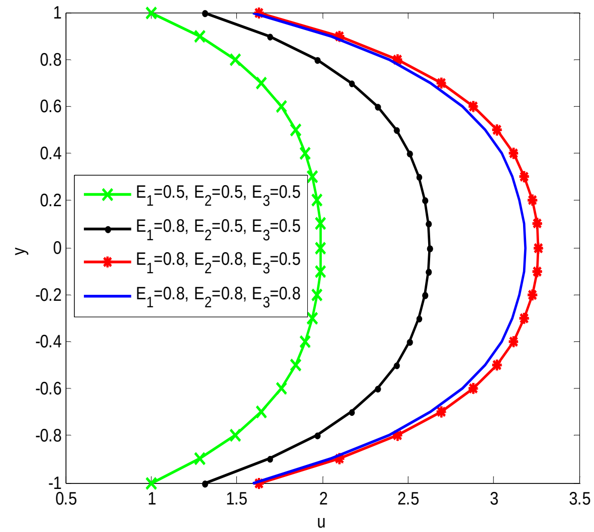


Fig.8. The variation of u with y for different values of E_1 , E_2 and E_3 for fixed ε=0.1, β =0.2, *M*=2, *Da*=2, *m*=0, $\lambda_1 = 1$.

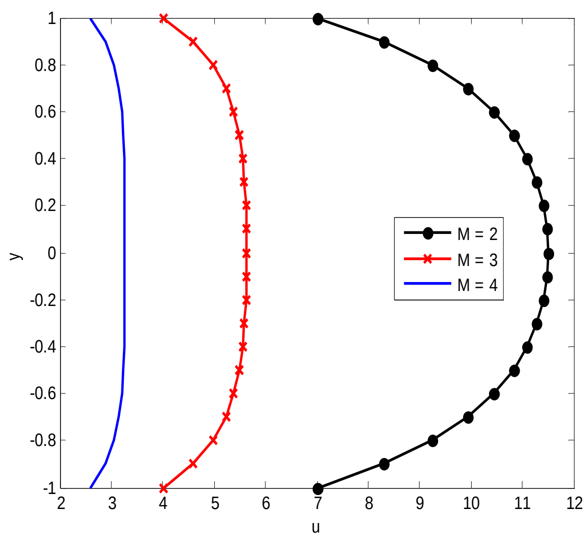


Fig.7. The variation of u with y for different values of *M* for fixed E_1=2, E_2=0.7, E_3=0.1, ε=0.2, β =0.2, *Da*=2, *m*=0.1, λ_1 =1.

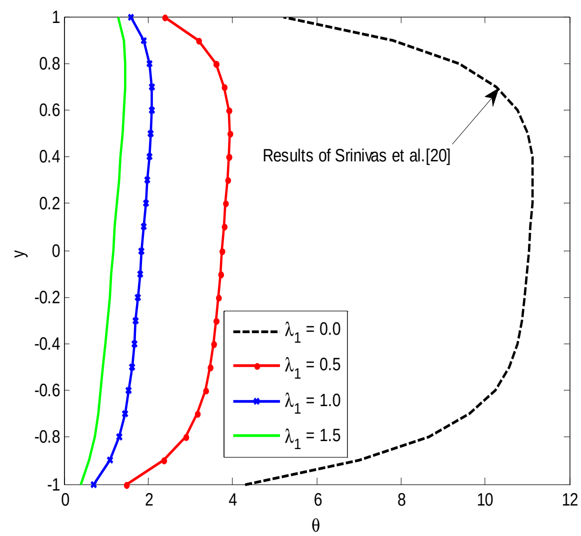


Fig.9. The variation of θ with y for different values of λ_1 for fixed *Br*=3, E_1=0.8, E_2=0.5, E_3=0.2, ε=0.1, β =0.1, *M*=2, *Da*=1, *m*=0.2.

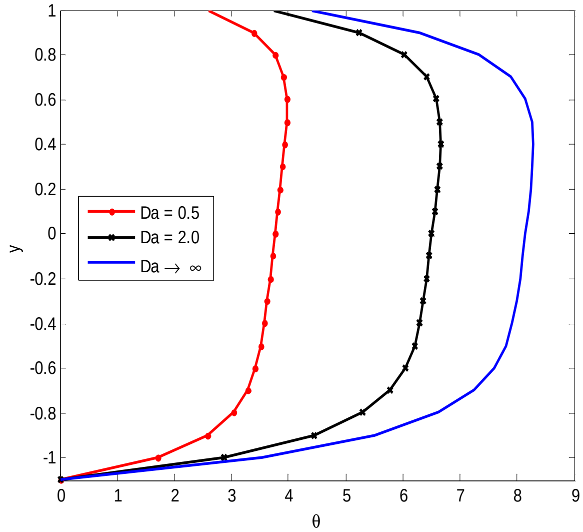


Fig.10. The variation of θ with y for different values of *Da* for fixed *Br*=4, E_1=1, E_2=0.5, E_3=0.2, ε=0.1, *M*=2, *m*=0.2, β =0.1, λ_1 =1.

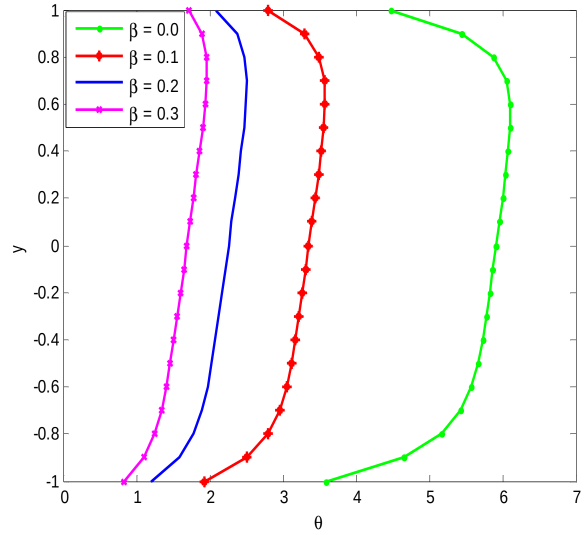


Fig.12. The variation of θ with y for different values of β for fixed *Br*=3, E_1=0.7, E_2=0.5, E_3=0.2, ε=0.2, *M*=3, *Da*=2, *m*=0.1, λ_1 =0.5.

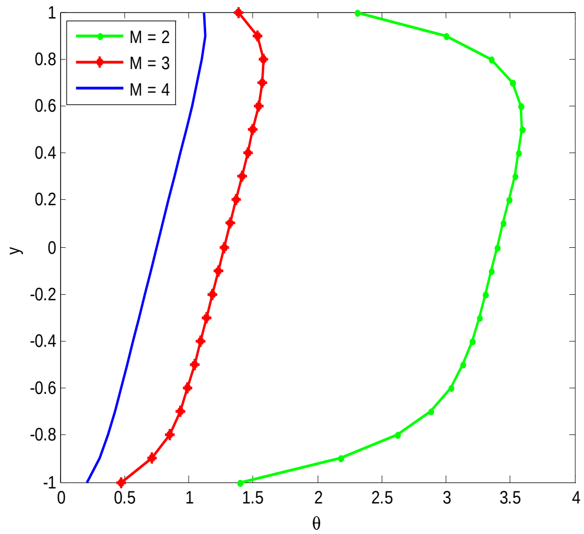


Fig.11 The variation of θ with y for different values of *M* for fixed *Br*=2, E_1=1, E_2=0.8, E_3=0.2, ε=0.1,*Da*=0.5, *m*=0.2, β =0.1, λ_1 =0.5.

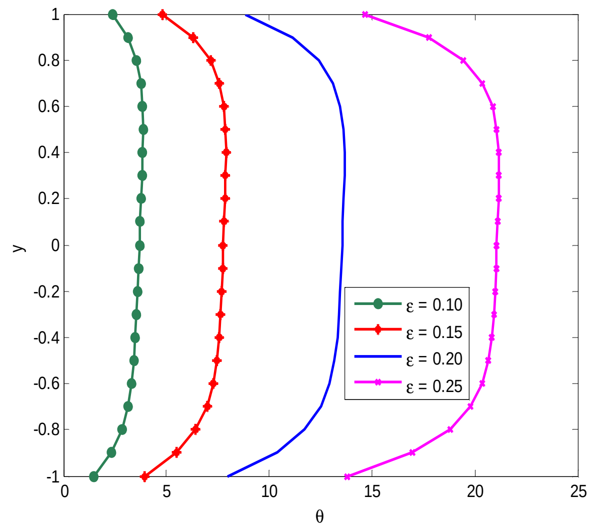


Fig.13. The variation of θ with y for different values of ε for fixed *Br*=3, E_1=0.8, E_2=0.5, E_3=0.3, λ_1 =1, β =0.1, *M*=2, *Da*=1, *m*=0.2

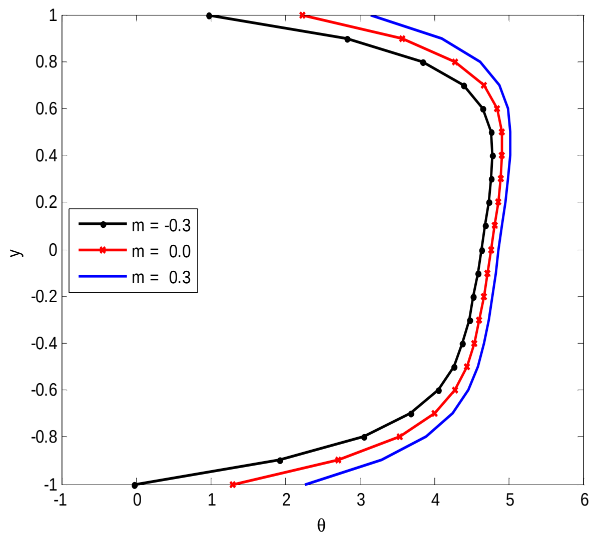


Fig.14. The variation of θ with y for different values of m for fixed *Br*=4, E_1=0.8, E_2=0.5, E_3=0.2, ε=0.1, *M*=2, *Da*=1, β =0.1, λ_1 =0.5.

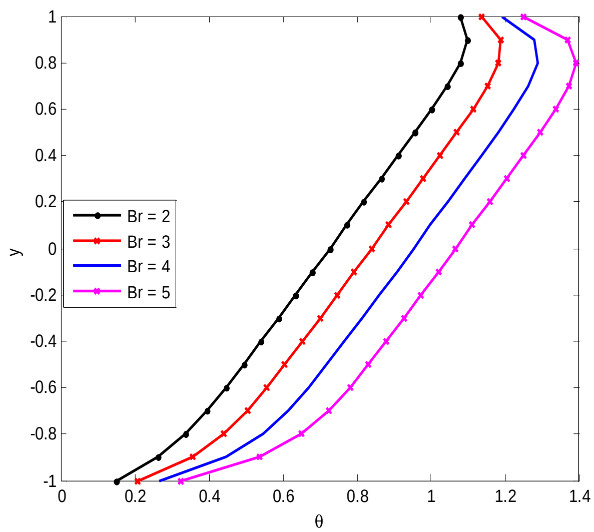


Fig.15.The variation of θ with y for different values of *Br* for fixed E_1=0.8, E_2=0.6,E_3=0.2, λ_1 =1, β =0.1, *M*=3, *Da*=1, *m*=0.1, ε =0.1

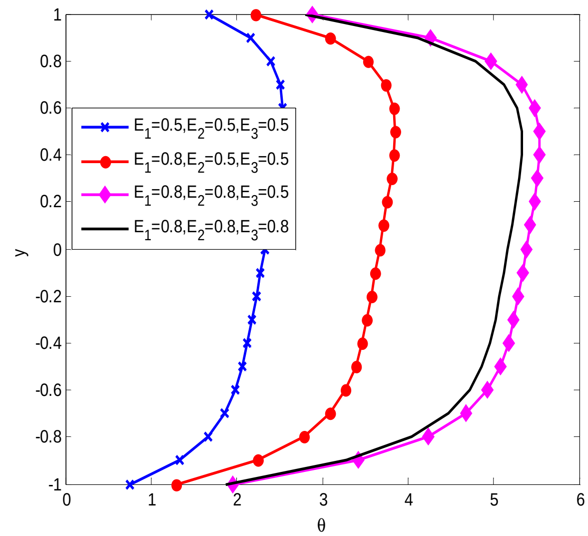


Fig.16. The variation of θ with y for different values of E_1, E_2 and E_3 for fixed *Br*=3, ε=0.1, β =0.1, *M*=2, *Da*=2, *m*=0.1, $\lambda_1 = 1$.

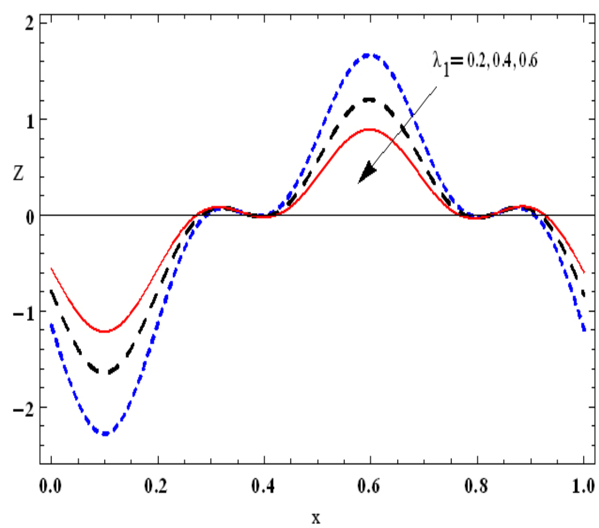


Fig. 17. The Coefficient of heat transfer Z with x for different values of λ_1 for fixed E_1=0.5, E_2=0.4, E_3=0.1, ε=0.1, *M*=3, *Br*=2, *Da*=0.2, *m*=0.1, β =0.1.

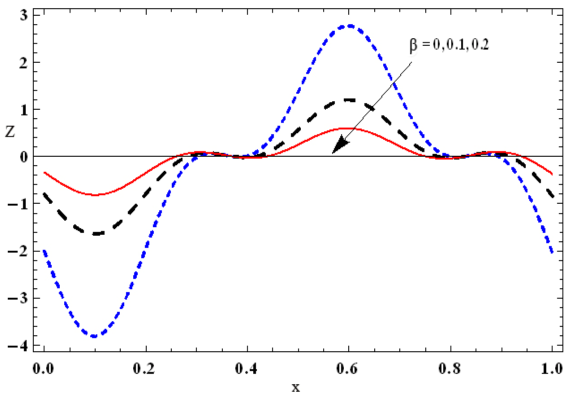


Fig. 18. The variation of Z with x for different values of β for fixed E_1=0.5, E_2=0.4, E_3=0.1, ε=0.1, M=3, Br=2, Da=0.2, m=0.1, λ_1 =0.4

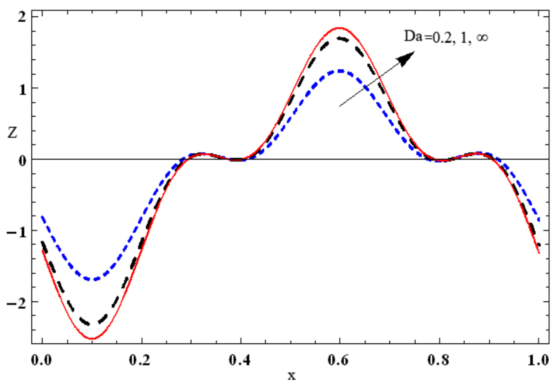


Fig.19. The variation of Z with x for different values of Da for fixed E_1=1.2, E_2=0.1, E_3=0.1, ε=0.1, M=5, Br=2, m=0.1, β =0.1, λ_1 =0.2.

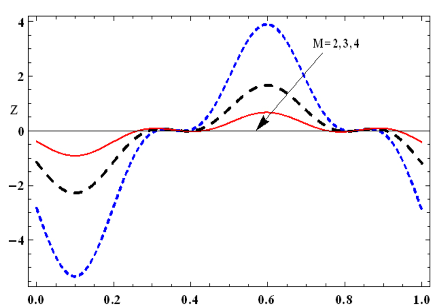


Fig. 20. The variation of Z with x for different values of M for fixed E_1=0.5, E_2=0.4, E_3=0.1, ε=0.1, Br=2, Da=0.2, m=0.1, β =0.1, λ_1 =0.2.

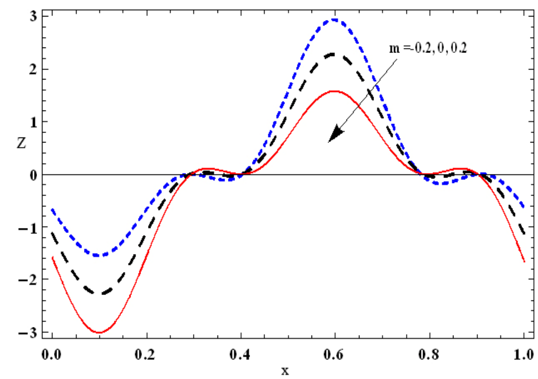


Fig.21. The variation of Z with x for different values of m for fixed E_1=0.5,E_2=0.4,E_3=0.1, ε=0.1,Br=3,Da=0.2,M=0.1, β =0.1, λ_1 =0.4.

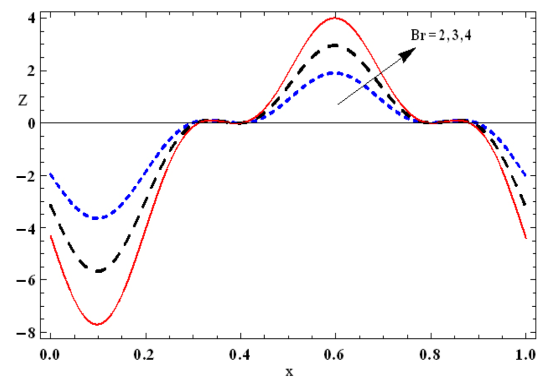


Fig. 22. The variation of Z with x for different values of Br for fixed E_1=0.8,E_2=0.4,E_3=0.2, ε=0.1,M=3,Da=0.2,m=0.2, β =0.1, λ_1 =0.4.

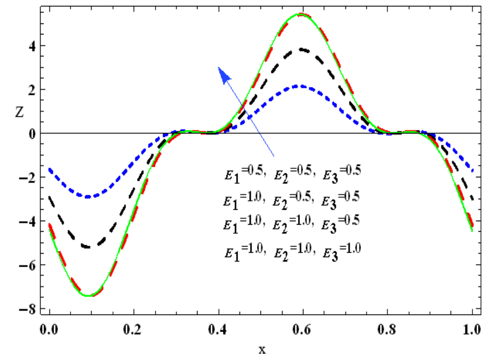
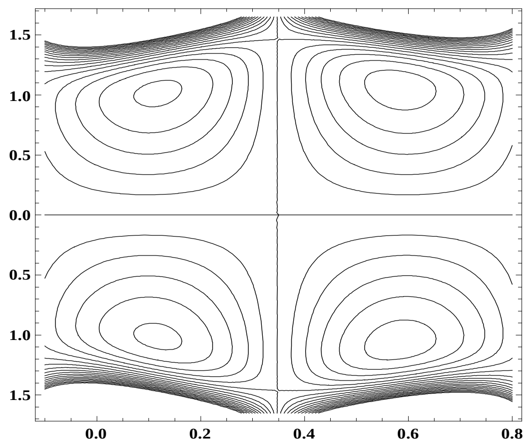
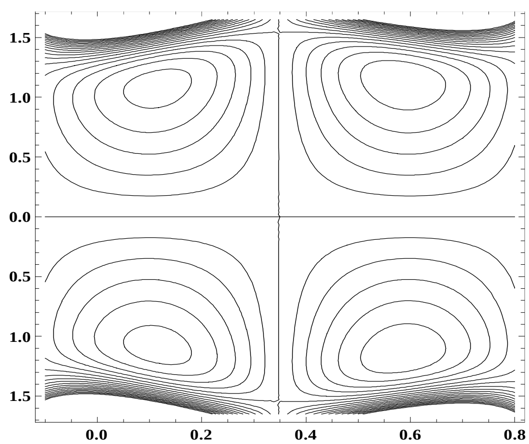


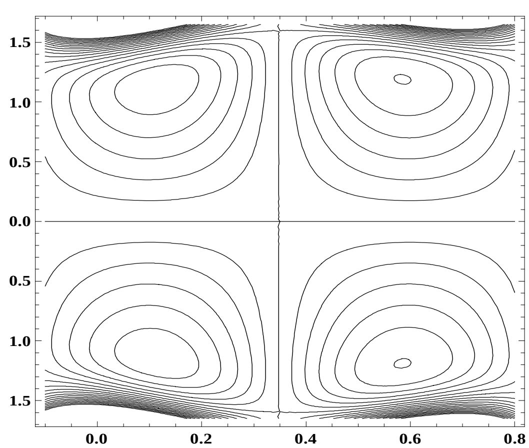
Fig. 23. The variation of Z with x for different values of E_1, E_2, E_3 for fixed M=3, Br=3, Da=0.2, m=0.2, β =0.1, λ_1 =0.4.



(a)

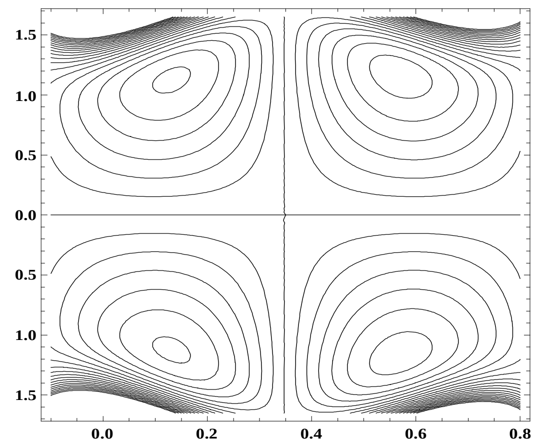


(b)

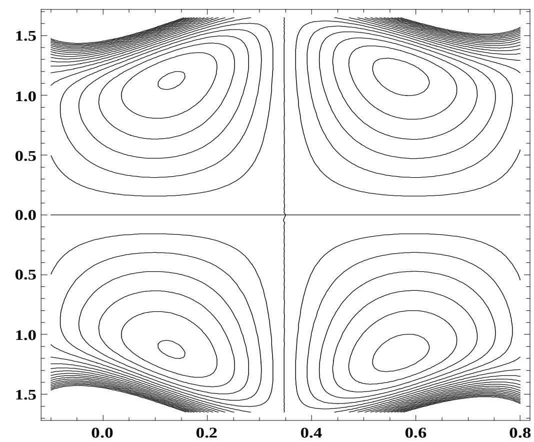


(c)

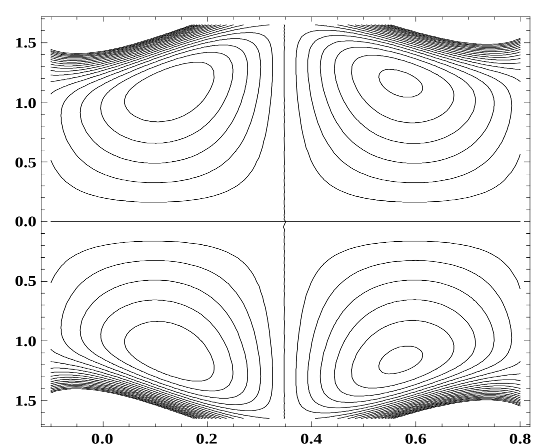
Fig. 24. Streamlines for (a) β =0, (b) β =0.1, (c) β =0.2, with E_1=0.6, E_2=0.4, E_3=0.1, ε =0.1, m=0.1, M=4, Da=0.1, λ_1 =1, t=0.1.



(a)

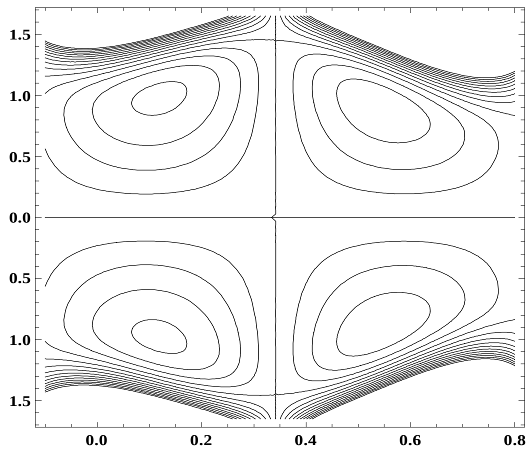


(b)

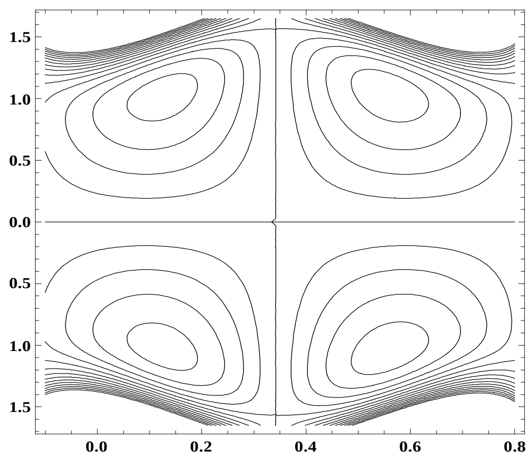


(c)

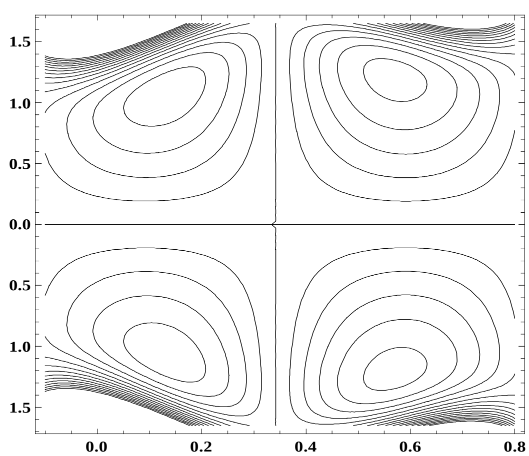
Fig. 25. Streamlines for (a) λ_1 =0, (Results of Srinivas et al.[20]) (b) λ_1 =0.2, (c) λ_1 =0.4, with E_1=0.6, E_2=0.4, E_3=0.1, ε =0.2, m=0.1, M=4, Da=0.05, β =0.1, t=0.1.



(a)

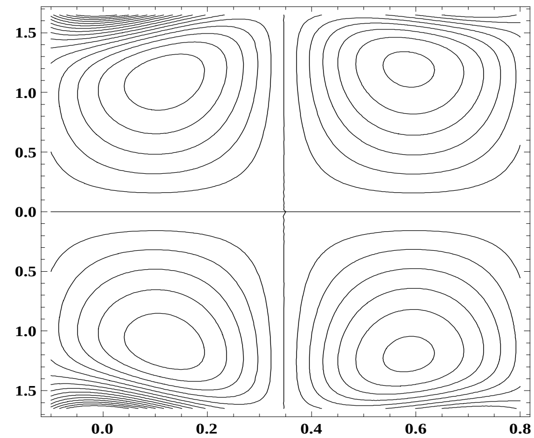


(b)

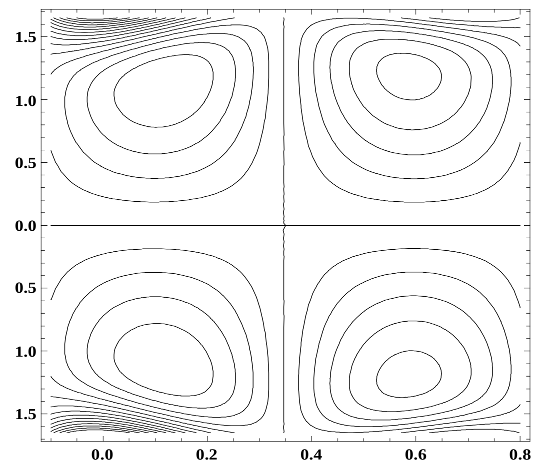


(c)

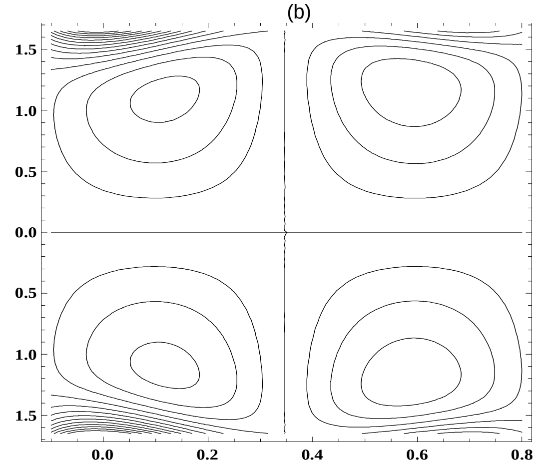
Fig. 26. Streamlines for (a) m=-0.3, (b) m=0.0, (c) m=0.3 with E_1=0.5, E_2=0.1, E_3=0.2, ε=0.2, β=0, M=4, Da=0.1, λ_1=1, t=0.1.



(a)



(b)



(c)

Fig. 27. Streamlines for (a) M=0, (b) M=1, (c) M=2 with E_1=0.8, E_2=0.7, E_3=0.2, ε=0.1, β=0.1, m=0.2, Da=0.1, λ_1=1, t=0.1.

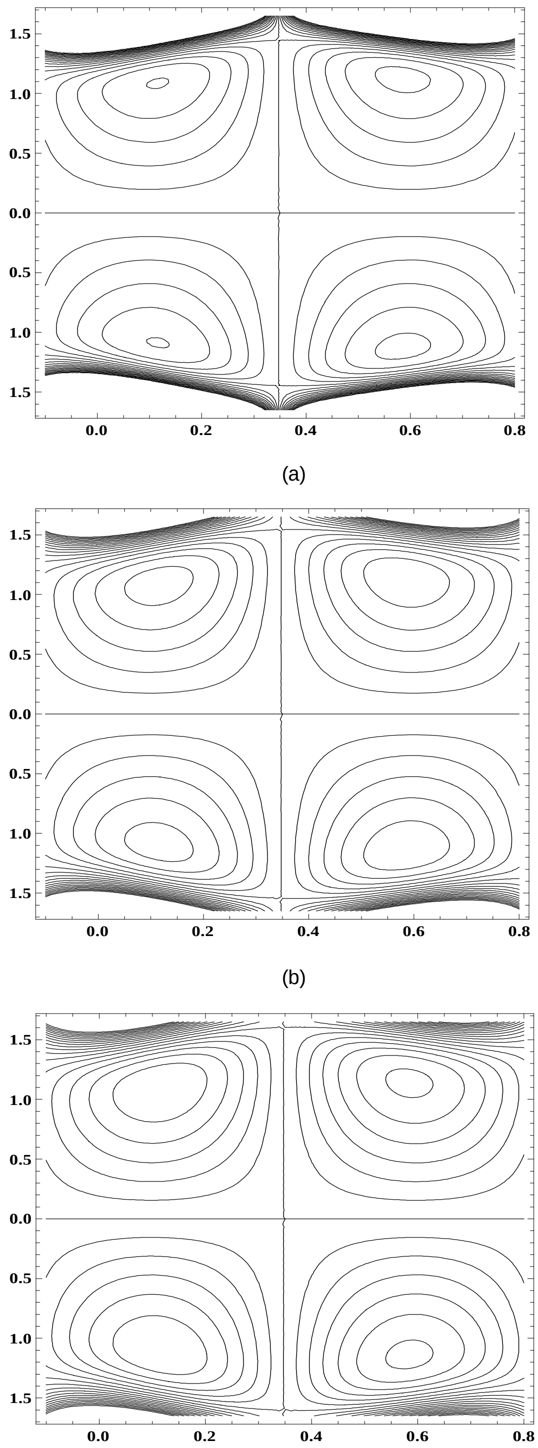


Fig. 28. Streamlines for (a) *Da*=0.01, (b) *Da*=0.1, (c) *Da*= ∞ with E_1=0.6, E_2=0.4, E_3=0.1, ε =0.1, β =0.1, *m*=0.1, *M*=4, λ_1 =1, t=0.1.

CONCLUSION

In this paper, we investigated the Influence of slip conditions, wall properties and heat transfer on MHD Peristaltic transport of a Jeffrey fluid in a non-uniform porous channel under the assumptions of long wavelength and low Reynolds number. The analytical expressions are obtained for the velocity, stream function and temperature. The main observations of this study are as follows:

- The velocity profile decreases with an increase in Jeffrey parameter λ_1.
- The temperature decreases with an increase in λ_1, β, *M* and E_3 while it increases with increase in *Da*, ε, *m*, E_1 and E_2.
- As expected, the coefficient of heat transfer is oscillatory in nature.
- The size of trapped bolus is smaller in Jeffrey fluid when compared with that of Newtonian fluid $(\lambda_1 = 0)$.
- As the Jeffrey parameter $\lambda_1 \to 0$, the results deduced are found to be in agreement with the corresponding ones of Srinivas et al. [20].

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